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THE IMPACT OF VARYING WELLBORE AREA ON FLOW SIMULATION

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ABSTRACT

Geothermal wellbore simulation involves solving coupled equations that express the conservation of mass, momentum and energy, for single or two-phase flow within the well. The usual derivation of the momentum flux term in the momentum equation assumes constant cross-sectional area, and is incorrect across changes in casing size. The correct cross-section averaging is reviewed, and it is shown that the momentum flux divergence is approximately a factor of two in error, if the constant area form is used. The effect of this error on simulated down-hole values and on simulated output curves is briefly examined.

WELLBORE SIMULATION EQUATIONS

The equations governing the flow of steam and liquid water in a well have been the subject of considerable effort over the years. Detailed two-fluid derivations in three dimensions give the most fundamental forms (e.g. Ishii 1975, Ishii & Hibiki 2006, Lahey & Drew 1988), and these are averaged over ensembles, and then across the cross-section of the well, to obtain one-dimensional equations for each phase. These may be added to reduce to equations for water mass conservation, and overall momentum and energy conservation. Then empirical correlations are required to relate steam saturation to vapor and liquid phase velocities.

Mass Conservation

Conservation of water mass may be expressed after averaging over cross-section area in the form (e.g. Taitel 1994, Chisholm 1983)

$$G \equiv \ [S\, \rho_v u_v + (1-S)\rho_l u_l]\, Q/A \qquad (1)$$

where S is steam saturation, ρ is density, u is velocity, Q is the total mass flow rate of water (kg/s), A is the cross-sectional area which can vary with depth z, and subscripts v and l refer to vapor and liquid phases resp. Q is constant unless there is a feed.

Momentum Conservation

Momentum conservation for water flow in a well with possibly varying A can be written, after averaging over cross-section area and adding together the vapor and liquid conservation equations, in the form (e.g. Taitel 1994, Yadigaroglu & Lahey 1976)

$$\frac{dP}{dz} = -G\,\frac{du_m}{dz} + \frac{f}{4r}\rho_e u_e |u_e| + \rho_m\, g\ \sin\theta \qquad (2)$$

where $\boldsymbol{P}$ is pressure, $\boldsymbol{r}$ is effective wellbore radius, $\boldsymbol{u_m}$ is the flowing average velocity

$$u_m = X\, u_v + (1-X)u_l$$

and X is the flowing steam quality (mass flowrate of vapor phase divided by Q), f is the Darcy friction factor with a two-phase correction included, ρ_e is an effective density (e.g. Chisholm 1983), u_e is an effective velocity (positive for upwards flow or production), ρ_m is the average density $S\rho_v + (1-S)\rho_l$, g is gravitational acceleration, and θ is the angle of the current wellbore section to the horizontal. Further details on the derivation of (2) may be found in the Appendix.

Energy Conservation

Conservation of energy takes the form (e.g. Chisholm 1983, Yadigaroglu & Lahey 1976)

$$\frac{d}{dz}(h + E_k) + g\ \sin\theta = \ Q_e\ , \qquad (3)$$

where the average flowing enthalpy is

$$h = Xh_v + (1-X)h_l$$

and h_v, h_l are the enthalpies of vapor and liquid phases of water, respectively. $E_k = \frac{X}{2}\,{u_v}^2 + \frac{(1-X)}{2}{u_l}^2$ is a flowing kinetic energy, and Q_e accounts for flow of heat to or from the surrounding country. No assumption of constant area is required to derive this energy conservation equation, it depends on area correctly through the flowing steam quality X.

VARYING WELLBORE CROSS-SECTION

The above equations are correct for varying area $A(z)$. They agree with all of the terms in Chisholm's (1983) conservation equations except for the divergence of the momentum flux term $G\frac{du_m}{dz}$, also termed the acceleration gradient. Since this is the only place that there is some question about which is the correct form to use when area varies, there is a detailed derivation in the Appendix.

We are dropping the minus sign for simplicity in the following. Taitel (1994) has the same term, correct for varying area; Chisholm's work however assumes constant area and is correct only in that case. Barelli *et al* (1982) and the simulator GWELL (1991) (Bjornsson, 1987) use Chisholm's form, which is

$$\frac{d}{dz}(Gu_m).$$

The error made in using Chisholm's form is of the same order as the size of the correct momentum flux divergence term. This follows by considering $G = \rho_e u_m$ which essentially defines the effective density ρ_e, and ignoring changes in effective density across an area change compared with changes in velocity,

$$\frac{d}{dz}(Gu_m) = \frac{d}{dz}(\rho_e {u_m}^2)$$
$$= 2\rho_e u_m \frac{d}{dz}(u_m) = 2G\frac{d}{dz}(u_m)$$

so that using Chisholm's form gives a value for the acceleration term that is twice the correct value. This result is also borne out by numerical simulations. It is only an issue across changes in casing diameter – when area is constant, both forms for the acceleration term are equivalent, and the above analysis fails due to the relative importance of changes in effective density.

Another form that is sometimes used for the acceleration term is

$$G\frac{d}{dz}(\,Gv_e)$$

where $v_e \equiv {}^1\!/_{\rho_e}$ is an effective specific volume (see the Appendix). This form is correct for variable area.

Using the incorrect form

$$\frac{d}{dz}(\,G^2 v_e)$$

again gives a value that is about twice the correct momentum flux term, if density is assumed constant across the change in area.

An area reduction means a velocity increase which means the divergence of momentum flux gives a drop in pressure gradient. Using the incorrect form overestimates this drop, that is, gives larger pressure drops across an area reduction than the correct value. This is what we observe in simulations also: computing top-down from a fixed wellhead pressure, the bottom-hole pressure is always a little higher when using the correct formulation.

As noted by Barelli *et al* (1982) the momentum flux term is often negligible. However, it can be important at wellbore locations where the casing radius changes dramatically, due to rapid deceleration there. It is difficult to make any general statements about the relative importance since gravity and friction play important roles in determining the momentum balance, and a geothermal wellbore simulator will try to find values of temperature and static steam fraction (in a two-phase flow) that give conservation of mass, momentum and energy up or down a well to within some accuracy. The best way to assess the impact of using the incorrect form on overall down-hole properties and on output curves, is to simulate numerically. The simulator *Simgwel* (based on GWELL but extensively modified, for example correcting GWELL's incorrect coding confusion of flowing steam quality and static dryness) is used in the following examples, to explore this. The maximum effect observed on bottom-hole pressure, over the ranges simulated, is 0.6 bara (see Table 1).

NUMERICAL SIMULATIONS

Numerical simulations with *Simgwel* have been conducted for a model vertical wellbore with casing radius 0.1m down to 500m, and 0.0707m from 500-1500m, so that area halves at 500m. Roughness is set to 0.00457m and no heat transport is allowed between casing and country. Orkiszewski's correlation (e.g. Chisholm 1983) is used in two-phase regions. Simulations were run to compare correct and incorrect treatment of the change in wellbore cross-sectional area at 500m depth. Table 1 shows the results for various flowrates Q and wellhead enthalpies, covering liquid, low enthalpy, high enthalpy and steam flows. The simulations were first run top-down, with wellhead pressure set to 8 bara. The effects on bottom-hole wellbore pressures of using the incorrect area formulation are shown. The effect increases with flowrate (which increases G and with enthalpy (which increases u_m). Using the incorrect form always reduces bottomhole pressure in these topdown simulations, as expected.

Calculations were also made (see Table 1) of the percentage error in the momentum flux gradient computation, as a percentage of the correct momentum flux gradient, and as a percentage of the correct total pressure gradient, at the change in area

at 500m. The relative importance of the momentum flux term for calculating pressure gradient can be calculated from Table 1 by dividing the percentage error of the total pressure gradient by the percentage error of the momentum flux term. It ranges from as low as 0.5% for slow liquid to as much as 720% for a high enthalpy flow. The table shows that the momentum flux is the dominant term at places where the area halves, for two-phase flows. For liquid or vapor flow, gravity still dominates.

Table 1: The simulated effect of incorrect momentum flux terms on bottom-hole pressures, and on pressure gradient at 500m when area halves there, for various flowrates Q and wellhead flowing enthalpies h. The correct bottom-hole pressure P bott (bara) is given, and the increase in bottom-hole pressure ΔP that results from using the incorrect momentum flux. The percentage errors shown are the calculated errors in the momentum flux as a percentage of the correct momentum flux term, and of the total pressure gradient, at 500m. The Orkiszewski correlation was used.

Q (kg/s)	H (kJ/kg)	P bott	ΔP	% err mom	% err total
20	400	146.411	0.004	100	0.5
40	400	149.46	0.02	100	2
80	400	161.52	0.07	100	8
20	1200	35.30	0.07	85	105
40	1200	56.2	0.3	89	370
80	1200	108.4	0.6	93	450
20	1800	43.1	0.1	101	720
40	1800	71.9	0.3	92	380
80	1800	131.4	0.5	93	320
1	2765	9.207	0.003	100	47

Table 1 shows that the approximate calculation above (which indicates a factor of two difference between the correct and incorrect area treatments) is quite accurate, since the error as a percentage of the correct acceleration term ranges from 85% to 101%. Combined with the relative importance of the acceleration term at the location where the area changes, this means the pressure gradient at changes in casing size is quite sensitive to whether the correct momentum flux formulation is used or not.

Nevertheless, since there is only one location at which area changes, the net effect overall, as measured say by the bottom-hole pressure when iteration from the wellhead downwards, is too small to be noticeable in pressure, temperature or velocity plots, at up to only 0.5%.

Down-hole pressures and mass-weighted velocities u_m computed topdown, comparing correct with incorrect area treatment, are shown in Fig. 1 for the worst case studied in Table 1, when $\mathrm{Q} = 80$ kg/s and $\mathrm{h} = 1200$ kJ/kg. In Fig. 1 the difference between symbols (correct values) and lines (incorrect values) is barely discernable on the scale of the full well depth range.

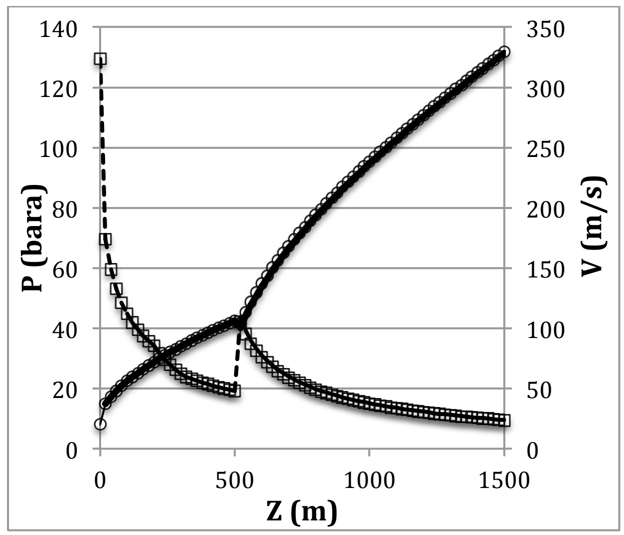

Figure 1: Simgwel down-hole values of pressure (circles and solid line) and velocity (m/s, squares and dashed line) versus depth for $Q = 80$ kg/s and $h = 1200$ kJ/kg. Symbols show correct results, lines show results using incorrect acceleration term.

The zoom-in in Fig. 2 shows the difference is only noticeable on the scale of the 20m node spacing at 500m.

The top-down simulations at 8 bara, together with given values for reservoir pressure at 1500m, give inferred values for the productivity index at bottom-hole, which are affected by the differences in bottom-hole pressures computed when comparing correct with incorrect area treatment. These are then used to compute output curves using bottom-up simulations as shown in Figure 3.

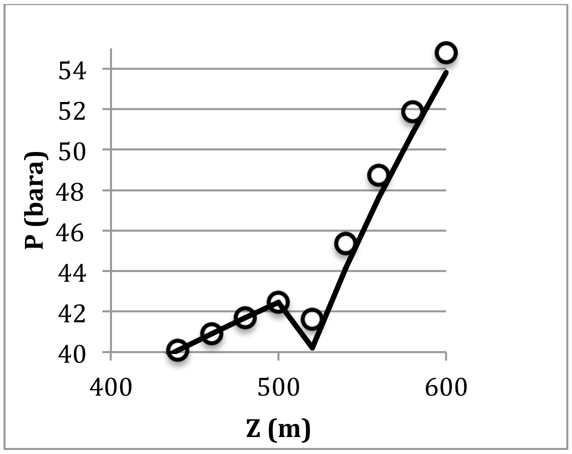

Figure 2: A zoom-in on part of the pressure data vs depth in Fig. 1, near the area change at 500m. Symbols show correct results, and the line shows incorrect results giving too big a pressure drop at 500m.

More realistic perhaps would be to do a top-down simulation at lower flowrates and higher wellhead pressures, possibly to match down-hole data. Hence top-down simulations were also run using 30 bara wellhead pressure and 15 kg/s flowrate, and output curves simulated using the different productivity indices obtained. Figure 4 shows the resulting output curves.

These output curves all show only barely discernable differences between correct and incorrect area treatments. Incorrect treatments give slightly higher maximum flowrates, due to the slightly larger inferred feedpoint productivities obtained. This behaviour depends critically on how close the bottom-hole pressure is to the reservoir pressure, and on whether maximum flow is limited by the feed or by the wellbore.

CONCLUSIONS

Using the incorrect form of momentum flux causes it to be out by a factor of two, which can have a significant effect on the pressure gradient computed at a change in casing diameter, since for a two-phase flow that change generally leads to the contribution from momentum flux being dominant there. However, such changes only occur at a few places in a wellbore. This means that the effect of using incorrect momentum flux terms, on overall wellbore pressure, temperature and velocity profiles, and on output curves computed, is small.

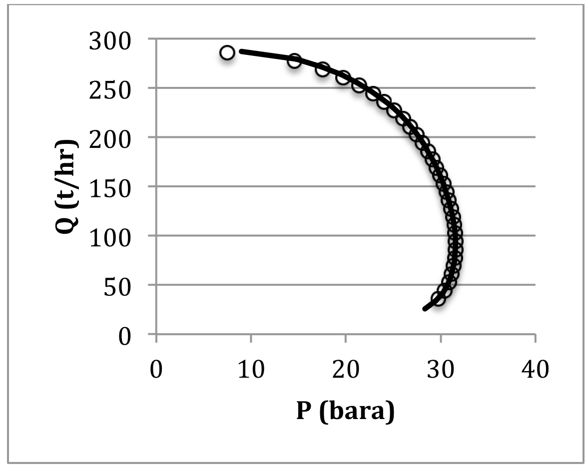

Figure 3: The output curves that result from down-hole simulations shown in Fig. 1 with WHP 8 bara, provided reservoir pressure is chosen to be 110 bara. Correct acceleration treatment results are the circles, lines are the result of using the incorrect area form.

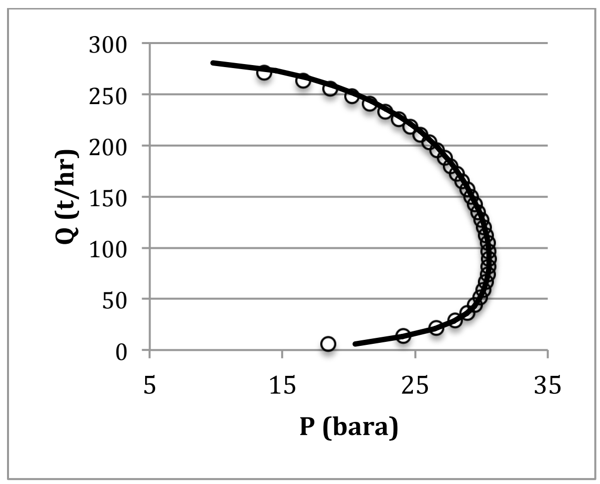

Figure 4: The output curves that result from a top-down simulation with WHP 30 bara and flowrate 15 kg/s, provided reservoir pressure is chosen to be 111 bara. Correct acceleration treatment results are the circles, lines are the result of using the incorrect area form.

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APPENDIX

Since there may be some confusion over what formulation is correct when wellbore area changes, here is an outline of the derivation of the momentum conservation equation (2), based on the work of Drew & Wood (1985), Ishii & Hibiki (2006), and Vij & Dunn (1986). A steady-state force balance on vapor and liquid phases separately, after taking an ensemble average in three dimensions, then integrating over the wellbore cross-section, gives

$$\frac{1}{A}\frac{\partial}{\partial z}(A\alpha_k\rho_k u_k^2) = \frac{1}{A}\frac{\partial}{\partial z}(A\alpha_k T_k) + M_k + u_{ki}{}^m\Gamma_k + \frac{4}{D}\alpha_{kw}T_{kw} + \alpha_k\rho_k g\sin\theta. \tag{A1}$$

The term on the left-hand side is the divergence of the flux of momentum, the first term on the right-hand side accounts for fluid stresses, M_k accounts for the vertical component of interfacial force density, Γ_k is the interfacial source of phase k where $k = v, l$ for vapor or liquid phase respectively, the fourth term accounts for wall stresses, and the final term for gravity. A is the cross-sectional area of the wellbore, and can vary with depth z. α_k is the static volume fraction of phase k, averaged over area. $u_{ki}{}^m$ is the velocity of phase k averaged for momentum at the interface between phases, and a correlation coefficient that accounts for the fact that the average of the square of velocity is not the same as the square of the average velocity has been set to one for simplicity (and can be absorbed into definitions later in any case). The subscript i refers to the interface between liquid and vapor phases, and the subscript w refers to the wall of the wellbore. The stress tensor for phase k is rewritten in terms of the pressure and the shear stress as $T_k = -p_k + \tau_k$. D is the hydraulic diameter of the wellbore.

The form of the momentum flux term already signals the fact that area will appear inside the derivative and its reciprocal is outside. At first glance then it seems that the pressure term that arises from the divergence of stress will also have this feature. However, the wall stress has a term in it that alters this behaviour to produce the usual $\frac{\partial P}{\partial z}$ term. This is described here.

Following the development on p. 47 of Drew & Wood (1985), it is useful to rewrite the wall stress term, using its definition

$$\alpha_{kw}T_{kw} = \frac{D}{4A}\oint \overline{\alpha_k}\,\overline{T_k^x}\!:\boldsymbol{n}\,\boldsymbol{e}_z\frac{dl}{\boldsymbol{n}.\boldsymbol{n}'} \tag{A2}$$

where the integral is around the perimeter of the well cross-section, over-bars indicate ensemble averages, $\boldsymbol{n}'$ is the two-dimensional unit normal in the cross-section, $\boldsymbol{n}$ is the unit normal to the wellbore curved surface, and $\boldsymbol{e}_z$ is the unit vector in the vertical direction.

This term arose from the area averaging process, applied to the stress term. The right-hand side can be expanded in terms of pressure and shear stress as (using results on pp. 41, 47 of Drew & Wood 1985)

$$\begin{aligned} -\frac{D}{4A}\oint \overline{\alpha_k}\,p_{kw}\boldsymbol{n}\cdot\boldsymbol{e}_z\frac{dl}{\boldsymbol{n}.\boldsymbol{n}'} + \alpha_{kw}\tau_{kw} &= -p_{kw}\frac{D}{4A}\left[\iint \frac{\partial\overline{\alpha_k}}{\partial z}\,dA - \frac{\partial(A\alpha_k)}{\partial z}\right] + \alpha_{kw}\tau_{kw} \\ &= \frac{Dp_{kw}}{4}\left[\frac{\alpha_k}{A}\frac{\partial A}{\partial z} + \frac{\partial\alpha_k}{\partial z} - \frac{1}{A}\iint \frac{\partial\overline{\alpha_k}}{\partial z}\,dA\right] + \alpha_{kw}\tau_{kw} \end{aligned} \tag{A3}$$

The double integrals are over the cross-sectional area. Assuming the gradient of the area averaged saturation α_k is close to the area average of the gradient of $\overline{\alpha_k}$, and that the area average at the wall α_{kw} is close to α_k, equations (A1) and (A2) simplify to

$$T_{kw} \approx \frac{Dp_{kw}}{4A}\frac{\partial A}{\partial z} + \tau_{kw} \tag{A4}$$

Now adding together the two equations (A1), one for each phase of water, using result (A4), and replacing $\alpha_v = S$, $\alpha_l = 1 - S$, the combined momentum equation is

$$\begin{aligned}\frac{1}{A}\frac{\partial}{\partial z}(AS\rho_v u_v^2 + A(1-S)\rho_l u_l^2) &= -S\frac{\partial p_v}{\partial z} - (1-S)\frac{\partial p_l}{\partial z} \\ &+ \frac{1}{A}\frac{\partial}{\partial z}(A[S\tau_v + (1-S)\tau_l]) \\ &+ \frac{4}{D}(\tau_{vw} + \tau_{lw}) \\ &+ (S\rho_v + (1-S)\rho_l)\,\mathrm{g}\sin\theta. \end{aligned} \tag{A5}$$

In this equation, the total interfacial source of momentum, which depends on capillary pressure, has been ignored. Then the pressures in liquid and vapor phase are equal, and shear stresses within the fluid are absorbed into an empirical correlation for the wall-induced shear stresses, so that eqn (A5) is simplified to

$$\frac{1}{A}\frac{\partial}{\partial z}(A[S\rho_v u_v^2 + (1-S)\rho_l u_l^2]) = -\frac{\partial p}{\partial z} - \frac{f}{2D}\langle \rho v|v|\rangle + \rho_m\,\mathrm{g}\sin\theta$$

where $\rho_m \equiv S\rho_v + (1-S)\rho_l$ and f is the Darcy or Moody friction factor. The brackets $\langle . \rangle$ indicate a suitable average, and v is some average two-phase velocity, but the friction term is usually dealt with by using a correlation of some kind.

The momentum equation is usually rearranged to take the form

$$\frac{\partial p}{\partial z} = -\frac{1}{A}\frac{\partial}{\partial z}(A[S\rho_v u_v^2 + (1-S)\rho_l u_l^2]) - \frac{f}{2D}\langle \rho v|v|\rangle + \rho_m\,\mathrm{g}\sin\theta\,,$$

which is a statement that pressure gradient is equal to the sum of a momentum flux term, a friction term and a gravity term. This derivation allows area to vary with depth.

The momentum flux term is usually rewritten, using the slip formula (e.g. Chisholm 1983):

$$K = \frac{u_v}{u_l} = \left(\frac{X}{1-X}\right)\left(\frac{1-S}{S}\right)\frac{\rho_l}{\rho_v}$$

where X is the flowing steam quality, steam mass flow rate $AS\rho_v u_v$ divided by total mass flow rate Q, to get

$$S\rho_v u_v^2 + (1-S)\rho_l u_l^2 = G u_m$$

where the total mass flow rate $Q = AG$ is constant. Hence the momentum flux term is

$$\begin{aligned}-\frac{1}{A}\frac{\partial}{\partial z}(A[S\rho_v u_v^2 + (1-S)\rho_l u_l^2]) &= -\frac{1}{A}\frac{\partial}{\partial z}(AGu_m) \\ &= -\frac{1}{A}\frac{\partial}{\partial z}(Qu_m) = -\frac{Q}{A}\frac{\partial}{\partial z}(u_m) \\ &= -G\,\frac{\partial}{\partial z}(u_m)\end{aligned}$$

which correctly accounts for the fact that A can vary with depth, but Q is constant.

Another form that is used is based on the fact that

$$S\rho_v u_v^2 + (1-S)\rho_l u_l^2 = G^2 v_e$$

where the effective specific volume is

$$v_e \equiv \left[\frac{X}{\rho_v} + \frac{K(1-X)}{\rho_v}\right]\left[X + \frac{1-X}{K}\right]$$

so that the momentum flux term may be written alternatively in the form

$$\begin{aligned}-\frac{1}{A}\frac{\partial}{\partial z}(A[S\rho_v u_v^2 + (1-S)\rho_l u_l^2]) &= -\frac{1}{A}\frac{\partial}{\partial z}(AG^2 v_e) \\ &= -\frac{1}{A}\frac{\partial}{\partial z}(QGv_e) = -\frac{Q}{A}\frac{\partial}{\partial z}(Gv_e) \\ &= -G\,\frac{\partial}{\partial z}(Gv_e)\,.\end{aligned}$$

Note that since G depends on depth through A, it cannot be brought outside the derivative unless A is constant.