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# Correct Energy Conservation in Geothermal Wellbore Simulation

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## Abstract

A fundamental error of interpretation of static steam dryness versus flowing steam quality in the coding of the simulator GWELL has been found, that means it does not correctly conserve energy or momentum. GWELL, or the energy and momentum subroutines from it, may have formed a basis for other geothermal wellbore simulator codes in existence, prompting this note on how to quickly check if your favorite geothermal wellbore simulator is actually conserving energy. The possible consequences of using a simulator that does not conserve momentum or energy as presumed, are also briefly explored.

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## Keywords:

Geothermal wellbore simulation, Energy conservation, Static versus flowing steam quality, Static versus flowing enthalpy, GWELL, coding errors, SwelFlo

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## 1. Introduction

A geothermal wellbore simulator is intended to numerically solve conservation of mass, momentum and energy equations in the steady-state, for flow of vapour and liquid phases of water up a geothermal well. This note is prompted by two discoveries, one made while working on Fortran code from the simulator GWELL, that the energy and momentum conservation subroutines in GWELL as coded in 1991 use static steam mass fraction  $X$  where they should be using the flowing steam quality  $X_{\text{flo}}$ , and hence GWELL does not actually conserve energy or momentum. The second discovery was that a commercially available geothermal wellbore simulator was giving very different enthalpies to the simulator *SwelFlo* (2013) written by the author, and that the difference could be explained by considering the possibility that the commercial simulator was also using static  $X$  where it should have been using flowing  $X_{\text{flo}}$  values.

Note that the manual for GWELL is correct in its presentation of these conservation laws. The error that was made is in the coding of GWELL, confusing static and flowing steam dryness values. Note also that information available for other wellbore simulators (for example, Garg and Pritchett (1978); Garg et al (20048); Gudmundsdottir et al (2013) )

has correctly phrased energy conservation equations in all of the cases that the author has seen, but there is not enough information published (for example, showing the variation of flowing enthalpy with depth from a simulation) to know if the coding of these simulators is correct or not. The only simulator this author has found to be in error is a commercially available simulator.

The purpose of this note is to give a quick and easy way to test the code of a geothermal wellbore simulator, to see whether energy conservation is being correctly implemented or not, for a simulation of two-phase up-flow, without access to the source code. We also consider the possible impact on reservoir management decisions of using the incorrectly coded GWELL routines.

## 2. Conservation Laws

The steady-state conservation laws for a two-phase flow, as described in Chisholm (1983), McGuinness (2013) and in the GWELL manual (Aunzo, 1990; Aunzo et al., 1991; Bjornsson, 1987), are as follows. Cross-sectional area  $A$  is taken to be constant in these equations, and flow is averaged over ensembles for turbulent flow and over representative elementary volumes,

then over the wellbore cross-sectional area, before setting time derivatives to zero. Conservation of mass in the absence of a feedpoint is given by

$$G = S\rho_v u_v + (1 - S)\rho_l u_l, \quad (1)$$

where  $G \equiv Q/A$  is the total mass flowrate per unit area,  $Q$  is the constant mass flowrate (kg/s), positive for production,  $S$  is saturation (the volume fraction occupied by vapour phase, in the static sense of a snapshot of the flow at a given time),  $u$  is actual velocity of a phase (m/s),  $\rho$  is the density of a phase, and subscripts  $v$  and  $l$  refer to vapour and liquid phase properties respectively.

Conservation of momentum is given by a combination of friction, divergence of momentum flux, and gravitational force terms, ignoring surface tension effects, to obtain the following form for the rate of change of pressure  $P$  down the well (Chisholm, 1983, p. 141)

$$\frac{dP}{dz} = \phi_{FL0}^2 \left[ \frac{dP}{dz} \right]_{FL0} - \frac{d}{dz}(Gu_m) + \rho_m g \sin \theta \quad (2)$$

where  $z$  is depth (measured distance along the wellbore from the wellhead),  $\phi_{FL0}^2$  is a multiplier that converts the liquid frictional pressure change term

$$\left[ \frac{dP}{dz} \right]_{FL0} = \frac{\lambda G |G|}{4\rho_l r}$$

to a two-phase frictional pressure change term,  $\lambda$  is the Darcy friction factor,  $r$  is wellbore radius,  $u_m = X_{\text{flo}}u_v + (1 - X_{\text{flo}})u_l$  is a weighted average flowing two-phase velocity,  $\theta$  is the angle that the wellbore makes with the horizontal, and  $\rho_m = S\rho_v + (1 - S)\rho_l$  is a two-phase density. The flowing steam quality is the steam mass flowrate  $Q_v = AS\rho_v u_v$  (kg/s) expressed as a fraction of total mass flowrate,

$$X_{\text{flo}} = \frac{Q_v}{Q} = \frac{S\rho_v u_v}{S\rho_v u_v + (1 - S)\rho_l u_l}.$$

Conservation of energy may be written as a balance between flowing enthalpy, kinetic energy, and potential energy, as (Chisholm, 1983, p. 141)

$$\begin{aligned} \frac{d}{dz} \left[ X_{\text{flo}} h_v + (1 - X_{\text{flo}}) h_l + X_{\text{flo}} u_v^2 / 2 + (1 - X_{\text{flo}}) u_l^2 / 2 \right] \\ = g \sin \theta, \end{aligned} \quad (3)$$

where  $h$  is specific enthalpy, and heat flow between wellbore and the surrounding country has been ignored for simplicity. In a simulation, the user can set the thermal conductivity of the country to zero, to replicate this.

Note that the static steam mass fraction or dryness  $X$  can be expressed in terms of flowing steam quality  $X_{\text{flo}}$  or the (static) saturation as

$$X = \frac{X_{\text{flo}} u_l}{X_{\text{flo}} u_l + (1 - X_{\text{flo}}) u_v} = \frac{S \rho_v}{S \rho_v + (1 - S) \rho_l} .$$

The flowing enthalpy of the two-phase flow may be defined as

$$h_{\text{flo}} = X_{\text{flo}} h_v + (1 - X_{\text{flo}}) h_l ,$$

whereas static two-phase enthalpy is

$$h = X h_v + (1 - X) h_l .$$

Flowing and static values of  $X$  and  $h$  are only equal when there is no slip between vapour and liquid phases. An average kinetic energy for the two-phase flow can be defined as  $E_k = X_{\text{flo}} u_v^2 / 2 + (1 - X_{\text{flo}}) u_l^2 / 2$ , so that energy conservation in the absence of external sources or sinks takes the form

$$\frac{d}{dz} [h_{\text{flo}} + E_k] = g \sin \theta . \quad (4)$$

Note that measurements of enthalpy at wellhead, for example by using the James lip pressure method, are measurements of flowing enthalpy. Also note that energy conservation at feed points is properly expressed in terms of flowing enthalpies to and from the feed zone. For a single phase fluid, static and flowing enthalpies are the same.

### 3. A Test for Energy Conservation

A close look at the Fortran coding of the energy, momentum, friction, choking, Armand correlation, and Orkizewski correlation subroutines of the simulator GWELL as it existed in 1991 reveals that static dryness  $X$  is being used in place of flowing steam quality  $X_{\text{flo}}$  in the conservation equations, the two-phase friction term, the calculation of when a flow is choked, and in the correlations. This is incorrect. Hence GWELL does not conserve energy or momentum, and does not have the correct friction, choking, or correlation calculations, unless the vapor and liquid phases are flowing at the same speed, as in mist flow or in homogeneous flow. The appendix lists the subroutines in GWELL that are affected by this bug.

Comparing differences recently in simulation results between *SwelFlo* and a commercially available geothermal wellbore simulator, revealed a mismatch in the change of simulated values of total fluid enthalpy down the well, prompting the question whether the commercial simulator was also using incorrect conservation

equations, possibly due to a shared provenance with GWELL. In any case, it would be useful and reassuring to be able to check that a simulator is correctly conserving energy, without access to the source code.

A simple test that the flowing enthalpy is being correctly used to calculate energy terms is to consider a vertical simulation in which the changes in kinetic energy are negligible, compared to the change in potential energy. Then equation (4) is approximated as

$$\frac{dh_{\text{flo}}}{dz} \approx g , \quad (5)$$

which integrated over a length  $L$  of wellbore gives the change in flowing enthalpy as

$$\Delta h_{\text{flo}} \approx gL . \quad (6)$$

This provides the test, as follows: run the simulator with no heat loss to or from the country, and look at a length, say  $L = 1000\text{m}$  of vertical well, with a flowrate that is not too large but which has a significant amount of slip between vapour and liquid phase flowrates, so that flowing and static enthalpies are quite different, but changes in the kinetic energy term are negligible. Then the change in flowing enthalpy  $\Delta h_{\text{flo}}$  should be dominated by the change in potential energy  $gL$ , being about  $9.8 \text{ kJ/kg}$  over  $1000\text{m}$ .

Hence, if a simulator returns static enthalpies for a two-phase upflow that change by an amount close to  $9.8 \text{ kJ/kg}$  over a depth of  $1000\text{m}$  (or  $980 \text{ J/kg}$  over  $100\text{m}$ ) at low flow rates and with no heat loss to the country, this is an indication that it is not correctly using flowing enthalpy, and is likely not conserving energy. The velocities of vapor and liquid phases need to be quite different for this test to distinguish between flowing and static enthalpy.

### 4. An Example

An illustration of the simple test is shown in Table (A.1). Three simulators are tested in topdown mode — *SwelFlo*, GWELL (Aunzo, 1990; Aunzo et al., 1991; Bjornsson, 1987) and a commercial one. The *SwelFlo* results show the expected result that flowing enthalpy changes by  $100g \approx 980 \text{ J/kg}$  over the depth of  $100\text{m}$ , matching the change in potential energy as given by the energy conservation equation, while static enthalpy change is significantly higher than this. Kinetic energy changes were observed to be less than  $10 \text{ J/kg}$ . GWELL, with its built-in confusion of flowing and static dryness, has the static enthalpy changing by  $980 \text{ J/kg}$ , and this is a reflection of the correctly stated but incorrectly coded

energy conservation subroutine. The wellhead enthalpy of 1141 kJ/kg given by the commercial simulator output could be either flowing or static enthalpy. However, the stated wellhead pressure of 27 barg and temperature of 229.16 at an elevation of 700m with XCO<sub>2</sub> of 0.38 weight% suggests that the enthalpy output by the commercial simulator is the static enthalpy. Disturbingly, then, the commercial simulator has the same change in static enthalpy as GWELL, which is using incorrect coding.

## 5. Consequences

The effect on downhole pressure of confusing static and flowing steam quality is in general highly dependent on the details of the simulation. Consequences are small at low concentrations of carbon dioxide, since pressure changes are then controlled by momentum conservation, which is dominated by the static gravitational term  $\rho_m g \sin \theta$ , and the differences in saturation do not make enough difference in average density to show up within graphical accuracy. At higher carbon dioxide concentrations however, the differences between static enthalpies obtained using correct versus incorrectly computed steam dryness, give significant differences in temperatures and velocities, because changing the saturation changes the temperature, if pressure and carbon dioxide concentration are fixed. On the other hand, if the correlation or setup is such that over most of the wellbore vapour and liquid phases move at similar speeds, little difference is expected when using the incorrect coding.

We explore briefly here just one example showing the possible consequences of using a simulator with incorrectly coded conservation laws, on the downhole pressure P, temperature T, and average fluid velocity V, and on output curves.

We consider a vertical well with smooth casing of radius 0.1m, 1000m deep, with a single bottomhole feed. Carbon dioxide concentration is 1% everywhere. We run a topdown simulation using *SwelFlo*, with correct treatment of flowing steam quality, and a wellhead pressure of 20 bara, production of 20 kg/s, at a flowing enthalpy of 1300 kJ/kg. The resulting static enthalpy at wellhead is then 1197 kJ/kg. Orkiszewski correlations are used in all simulations (Chisholm, 1983).

Note that topdown simulations have an advantage over bottom-up simulations: a steady-state solution always exists to the full depth of the well for topdown simulations (provided temperatures do not reach critical), whereas for a bottom-up simulation it may be the case that the solution ceases to exist partway up the

wellbore (e.g., Figures 4, 5 and 6 in Bilicki and Kestin (1982)).

The resulting enthalpy and wellbore pressure at bottomhole, assuming a reservoir pressure of 40 bara there, allow to calculate a productivity index and then to compute an output curve using bottom up simulations with various wellbore pressures at bottomhole, assuming the reservoir enthalpy there is the value calculated with the topdown simulation to get a flowing enthalpy of 1300 kJ/kg at the wellhead.

The results of this correct simulation are then used as data, to compare with the results of running the simulator in “GWELL mode”, that is, with correlations and conservation laws as originally coded in GWELL with its confusion between flowing steam quality and static dryness. The P, T and V values from three GWELL runs are shown together with the data in Fig. 1. The run labelled GWELL 1 has static enthalpy matching the flowing enthalpy of 1300 kJ/kg and all other wellhead properties unchanged from the correct simulation, the one labelled GWELL 2 has static enthalpy matching the static enthalpy of 1197 kJ/kg and all other wellhead properties unchanged from the correct simulation (which then gives the most accurate match to wellhead conditions but a poor match elsewhere), and GWELL 3 has had enthalpy, pressure and flowrate at the wellhead slightly adjusted by hand in an attempt to more closely match overall values of downhole P, T and V.

It can be seen from Fig. 1 that it is difficult to match all three of P, T and V with GWELL simulations using incorrect energy computation. The differences in downhole values can be significant for this case.

Then to illustrate the effect of using incorrect simulation on the output curve, the hand-fitted GWELL 3 topdown simulation is used to calculate a bottomhole productivity index and reservoir enthalpy, and these are used to calculate (still using GWELL conservation laws) an output curve. This is compared with the correct simulation output curve in Fig. 2. Note that all simulations use a twenty-one point trapezoidal rule to integrate the pseudopressure formulation that gives accurate feedpoint viscosity (McGuinness, 2014).

The output curves show only small differences in flowing enthalpy, but because the productivity index for GWELL 3 is about twice the correct value, the GWELL 3 output at maximum flowrate is about 40% more optimistic at 56 kg/s than the correct simulation result of 40 kg/s.

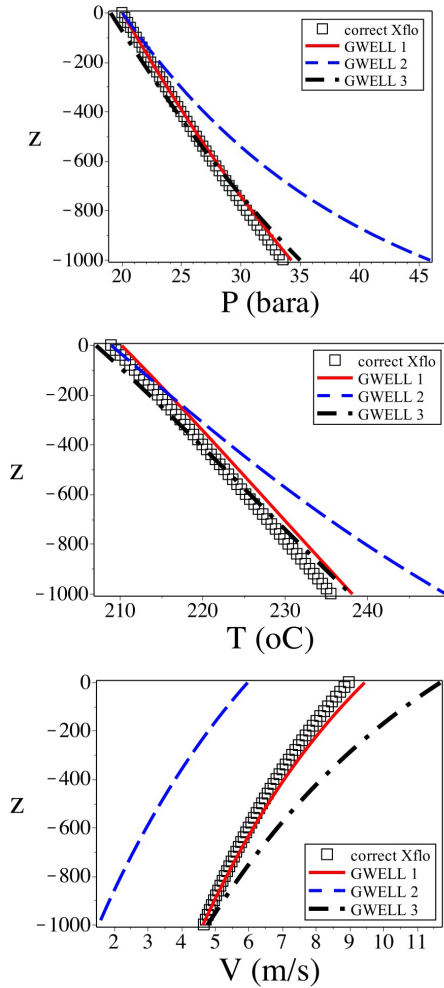


Figure 1. Downhole simulations run from the top down, showing  $P$ ,  $T$  and average fluid  $V$  versus depth (m). Squares are the correct simulation results, treated as data for the GWELL runs. Simulation GWELL 1 uses the correct wellhead flowing enthalpy as the wellhead enthalpy, and simulation GWELL 2 uses the static wellhead enthalpy as the wellhead enthalpy. Otherwise they use the same  $P$ ,  $Q$  values as the correct run. GWELL 3 adjusts all wellhead values in an attempt to obtain a better overall match to  $P$ ,  $T$  and  $V$  downhole.

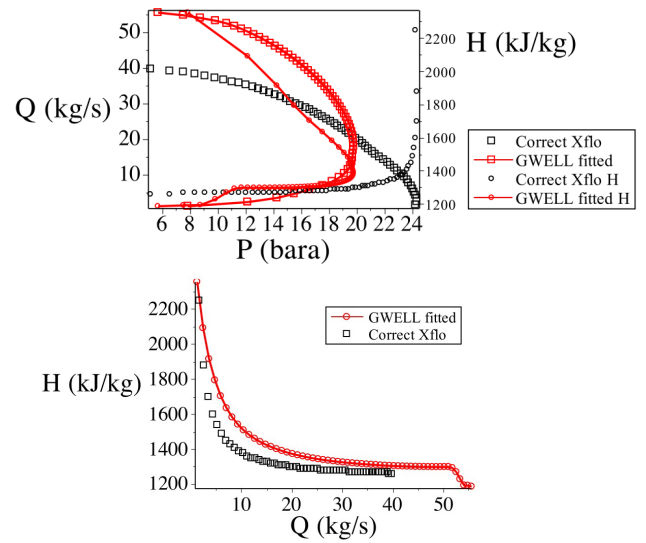


Figure 2. Output curves, showing wellhead flowrates  $Q$  versus wellhead pressure  $P$ , for the correct simulation and for the GWELL 3 simulation. The correct output curve was obtained by using bottom-hole pressure, flowrate, and enthalpy from the topdown simulation, to compute a productivity index. Also shown are the wellhead flowing enthalpies  $H$  computed, versus pressure in the first plot, and versus flowrate in the second plot.

## 6. Informed Decisions

Finally we explore a possible consequence of using a simulator with the incorrect coding seen in GWELL when making management decisions. For example, it might be desirable to simulate how much production from a geothermal well would be improved if wellbore radius was doubled to 0.2m.

With this aim in mind, the correct and GWELL 3 simulations were re-run with doubled radii, to obtain the results shown in Fig. 3. Both predict substantial improvements in the maximum flowrate, although the GWELL 3 simulation is much more optimistic with a flowrate that is nearly double that in the narrower well, whereas the accurate simulation forecasts a more conservative 50% improvement in flowrate. Both results indicate that the flow is wellbore limited, rather than feed productivity limited. This depends strongly on the productivity index, which with our choice of a 40 bara reservoir pressure in this model example is rather large.

## 7. Conclusions

An error of interpretation, so that static steam dryness was used in place of flowing steam quality in a number of subroutines affecting energy conservation, momentum conservation, and correlations, invalidates

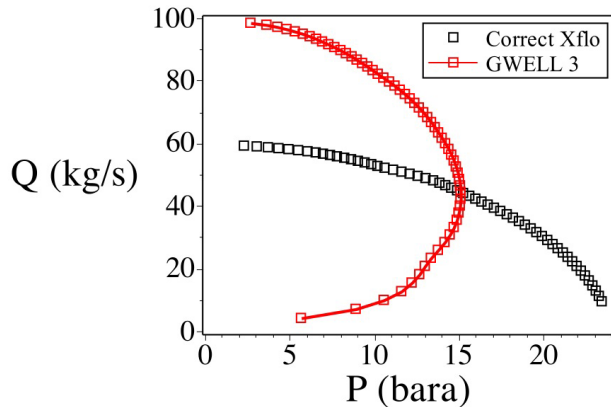


Figure 3. Output curves, with wellbore radius doubled to 0.2m, showing wellhead flowrates  $Q$  versus wellhead pressure  $P$ , for the correct simulation and for the GWELL 3 simulation.

conservation of energy and momentum to varying degrees in the geothermal simulator GWELL. This non-conservation can alter simulated downhole values of pressure, temperature and other fluid properties significantly, particularly when slip velocities are high and when carbon dioxide concentrations are high enough that errors in saturation affect pressures significantly.

Any simulator that is based on the 1991 version of GWELL might inherit this incorrect behaviour. A simple test, using a topdown simulation to check that flowing enthalpy changes by an amount roughly equal to  $g$  times depth, provides a litmus test for checking that the correct flowing steam quality is being used in the energy equation. A simulator that fails this litmus test might then be suspected of containing other confusions of steam quality, static dryness versus flowing quality.

Note that this litmus test can also be applied across a flashpoint — the flowing enthalpy above the flash depth should only differ from the flowing enthalpy below the flash point by  $gL$ . The flowing enthalpy in the liquid region below flash is of course the same as the static enthalpy. This provides a test that a simulator is faithfully conserving energy across a flashpoint, coping with the consequent change in dependent variables in going from pure liquid phase to two phase flow.

Getting the conservation equations wrong in this way has consequences for simulator outputs that are in general unpredictable. Our simple example shows that the consequences might have a significant impact on interpretation of downhole data, on anticipated output curves, and on management decisions based on what-if wellbore simulation scenarios.

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## Appendix A. GWELL subroutines affected

In GWELL, the variable  $X$  is used to denote both the static steam mass fraction solved for along with temperature in two-phase regions, and the flowing steam quality that appears in correlations, momentum conservation, energy conservation, friction calculations, and choking computations. The following GWELL subroutines are affected by the confusion between flowing steam quality and static steam fraction: ARMAND, CHOKED, ENERGY, FEED1, FEED2, FEED3, FRIC1, MOMENTUM, REGIME, RESMOM, and RESMOM2 (which also has the wrong sign on the momentum flux divergence term  $A1$ ).

The change required to correct can be nontrivial. For example, in ARMAND, the correlation essentially gives saturation in terms of flowing steam quality, which

when written correctly has to be iterated to get flowing properties given the current static properties. In GWELL this subroutine is written in a contradictory and confused way, by starting as if  $X$  is flowing not static steam mass fraction, then computing saturation from that through the correlation (but not properly iterating, just repeating twice and taking an average if necessary), and calculating a slip velocity where it should compute a drift velocity. So correction of this subroutine involves a complete rewrite.

depth	<i>SwelFlo</i>		GWELL		commercial
	$h_{\text{flo}}$	$h$	$h_{\text{flo}}$	$h$	$h$
0	1141.00	1063.4	1269.31	1141.00	1141.00
25	1141.25	1065.9	1265.30	1141.25	1141.25
50	1141.49	1068.3	1261.18	1141.49	1141.49
75	1141.74	1070.8	1256.95	1141.74	1141.74
100	1141.98	1073.4	1252.59	1141.98	1141.98

Table A.1. Enthalpy (kJ/kg) versus depth (m), from the first 100m of simulations using *SwelFlo*, GWELL, and a commercially available simulator.  $h$  is the static value of enthalpy, and  $h_{\text{flo}}$  is the flowing enthalpy (kJ/kg). Wellhead pressure is set to 27 bars gauge, enthalpy to 1141 kJ/kg, and flowrate to 19 kg/s. Weight % of CO<sub>2</sub> is set to 0.38. Orkiszewski's correlation is used in all simulations.